Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 21 March 2006 (MN LaTeX style file v1.4)

Starbursts near supermassive black holes: young stars in the Galactic Center, and gravitational waves in LISA band.

Yuri Levin[1,2]
[1] *Leiden Observatory, P.O. Box 9513, NL-2300 RA Leiden*
[2] *Lorentz Institute, P.O. Box 9506, NL-2300 RA Leiden*

printed 21 March 2006

ABSTRACT

We propose a scenario in which massive stars form in a self-gravitating gaseous disc around a supermassive black hole. We analyze the dynamics of a disc forming around a supermassive black hole, in which the angular momentum is transported by turbulence induced by the disc's self-gravity. We find that once the surface density of the disc exceeds a critical value, the disc fragments into dense clumps. We argue that the clumps accrete material from the remaining disc and merge into larger clumps; the upper mass of a merged clump is a few tens to a few hundreds of solar mass.

This picture fits well with the observed young stellar discs near the SgrA* black hole in the Galactic Center. In particular, we show how the masses and spatial distribution of the young stars, and the total mass in the Galactic Center discs can be explained. However, explaining the origin of the several young stars closest to the black hole (the S-stars) is more problematic: their orbits are compact, eccentric, and have random orientation. We propose that the S-stars were born in a previous starburst(s), and then migrated through their parent disc via type I or runaway migration. Their orbits were then randomized by the Rauch-Tremaine resonant relaxation.

We then explore the consequences of the star-formation scenario for AGN discs, which are continuously resupplied with gas. We argue that some compact remnants generated by the starburst will get embedded in the disc. The disc-born stellar-mass black holes will interact gravitationally with the massive accretion disc and be dragged towards the central black hole. Merger of a disc-born black hole with the central black hole will produce a burst of gravitational waves. If the central black hole is accreting at a rate comparable to the Eddington limit, the gas drag from the accretion disc will not alter significantly the dynamics of the final year of merger, and the gravitational waves should be observable by LISA. For a reasonable range of parameters such mergers will be detected monthly, and that the gravitational-wave signal from these mergers is distinct from that of other merger scenarios. Also, for some plausible black hole masses and accretion rates, the burst of gravitational waves should be accompanied by a detectable change in the optical luminosity of the central engine.

1 INTRODUCTION

It is widely believed that self-gravitating accretion discs can form around supermassive black holes (SBH) in AGNs. Theoretical models show that the AGN accretion discs must become self-gravitating if they extend beyond a fraction of a parsec away from a central black hole (Paczynski 1978, Kolykhalov and Sunyaev 1980, Schlosman and Begelman 1987, Kumar 1999, Jure 1999, Goodman 2003). Self-gravitating discs are unstable to fragmentation on a dynamical timescale; self-gravity of AGN accretion discs is a major issue in understanding how gas is delivered to the central black hole. It is likely that star formation will occur in the outer parts of an AGN accretion disc (Kolykhalov and Sunyaev 1980, Schlosman and Begelman 1987).

There are two lines of observational evidence that a starburst within SBH's radius of influence may be a common phenomenon:

(i) Levin and Beloborodov (LB, 2003), using preexisting data, have identified a disc of young massive stars which are moving clockwise in the gravitational potential of SgrA*, the SBH at the center of the Galaxy. They have argued that the stars were born in a dense accretion disc which existed several million years ago in the Galactic Center. The presence of the disc was later confirmed by Genzel et. al. (2003) and Paumard et al. (2005), who have used updated data sets; these authors also found a second counterclockwise disc of young massive stars. For the past few years, an alternative possibility for the origin of the young stars was considered on equal footing with LB'S proposal. In the alternative scenario (Gerhard 2001, Hansen and Milosavljevic 2003), the stars were born tens of parsecs from SgrA* and were originally members of a massive star cluster. This cluster then spiraled in towards SgrA* due to dynamical friction with an inner bulge, and deposited the stars near SgrA*. However, this scenario is now disfavored due to two recent observations: (a) In the cluster scenario one expects many young low-mass stars to also be present in the Galactic Center and for them to produce copious amounts of x-ray emission from their coronae. However, Chandra observations show that the Galactic Center is a relatively weak x-ray emitter, and thus the presence of a multitude of low-mass stars is ruled out (Nayakshin and Sunyaev, 2005); (b) Paumard et al. (2005) find no early-type stars outside of the central half-parsec, which again strongly argues against the in-spiraling cluster scenario. Therefore, most likely the young stellar disc formed in situ as result of fragmentation of a gaseous disc, as was argued by LB.

(ii) Bender et al. (2005) have used HST to identify a compact disc of A-stars which is located deep in the gravitational potential of a SBH in the nucleus of M31. The stars are likely much younger than the SBH (this is true unless one assumes that the SBH is older than $\sim 10^8$ years), and the strong tidal barrier makes it unlikely that the disc is a remnant of a tidally disrupted star cluster since any cluster would get disrupted at a larger distance from the SBH (Nayakshin, 2005). Therefore, the disc of A-stars is most likely the remnant of a gaseous accretion disc which existed in M31 about a hundred million years ago.

Star formation in an SBH radius of influence may be connected to supporting the high accretion of some AGNs (Thompson et al. 2005) and at the same time may help explain low luminocity of others (Tan and Blackman 2005). The dynamics of the fragmenting disc is strongly affected by the feedback energy input from the starburst. Collin and Zahn (1999) have conjectured that the feedback from this star formation will prevent the accretion disc from becoming strongly self-gravitating. However, Goodman (2003) has used general energy arguments to show that the feedback from star formation is insufficient to prevent an AGN disc with the near-Eddington accretion rate from becoming strongly self-gravitating at a distance of 10^4—10^5 Scwartzschild radii from the central black hole (about 0.1pc for our Galactic Center). This is distinct from the case of galactic gas discs, for which there is evidence that the feedback from star formation protects them from their self-gravity.

In this paper we concentrate on the physics of the self-gravitating disc and make a semi-analytical estimate of the possible mass range of stars formed in such discs (Sections 2 and 3). Our principal conclusion is that the stars can be very massive, up to hundreds of solar masses. This conclusion is in qualitative agreement with two recent independent observations: Nayakshin and Sunyaev (2005) and Paumard et al. (2005) have shown that the young stars in the GC must were produced in a starburst with the top-heavy IMF strongly favoring massive stars. In Section 4 we specialize to the case of SgrA* discs and show that the mass and the column density distribution of the marginally fragmenting disc are consistent with those of the currently observed stellar discs. We also address the puzzle of the several the young stars in the central arc-second (the S-stars). Their orbits present a problem for the disc-starburst picture. Their extreme proximity to SgrA*, eccentric orbits, and random inclinations exclude the possibility that they were born from the gaseous disc at their current location. We argue that instead they were born in a disc at a larger distance from SgrA*, but then migrated inwards due to gravitational interaction with the disc. Their eccentricity and inclination angles were randomized by relatively fast resonant relaxation, a process discovered by Rauch and Tremaine in 1996.

In section 5, we consider a self-gravitating AGN accretion disc which is continuously supplied by gas on a timescale greater than the lifetime of massive stars. Some of the black-hole remnants of the stars become embedded in the disc and and migrate inward on the timescale of $\sim 10^7$ years. The merger of the migrating black hole with the central black hole will produce gravitational waves. We show that for a broad range of AGN accretion rates the final inspiral is unaffected by gas drag, and therefore the gravitational-wave signal should be detectable by LISA. The rate of these mergers is uncertain, but if even a fraction of a percent of the disc mass is converted into black holes which later merge with the central black hole, then LISA should detect monthly a signal from such a merger. The final inspiral may occur close to the equatorial plane of the central supermassive hole and is likely to follow a quasi-circular orbit, which would make the gravitational-wave signal distinct from those in other astrophysical merger scenarios. If the disc-born black hole is sufficiently massive, it will disrupt accretion flow in the disc during the final year of its inspiral, thus making an optical counterpart to the gravitational-wave signal.

2 PHYSICS OF A FRAGMENTING SELF-GRAVITATING DISC

The importance of self-gravitating accretion discs in astrophysics has long been understood (Paczynski 1978, Lin and Pringle 1987). It was conjectured that the turbulence generated by the gravitational (Toomre) instability may act as a source of viscosity in the disc. This viscosity would both drive accretion and keep the disc hot; the latter would act as a negative feedback for the Toomre instability and would keep the disc only marginally unstable. Recently, there has been big progress in our understanding of the self-gravitating discs, due to a range of new and sophisticated numerical simulations (Gammie 2001, Mayer et. al. 2002, Rice et. al. 2003). In our analysis, we shall rely extensively on these numerical results.

Consider an accretion disc which is supplied by a gas infall. This situation may arise when a merger or some other major event in a galaxy delivers gas to the proximity of a supermassive black hole residing in the galactic bulge. Let $\Sigma(r)$ be the surface density of the disc. We follow the evolution of the disc as $\Sigma(r)$ gradually increases due to the infall.

We begin by assuming that initially there is no viscosity mechanism, like Magneto-Rotational Instability (MRI), to transport the angular momentum and keep the disc hot⋆. This assumption is valid when the ionization fraction of the gas in the disc is low, i.e. when the gas is far enough from a

⋆ When the disc begins to fragment, the viscosity due to self-gravity-driven shocks exceeds the one due to MRI; see below. Therefore, while computing the disc parameters at fragmentation, it is reasonable to ignore MRI

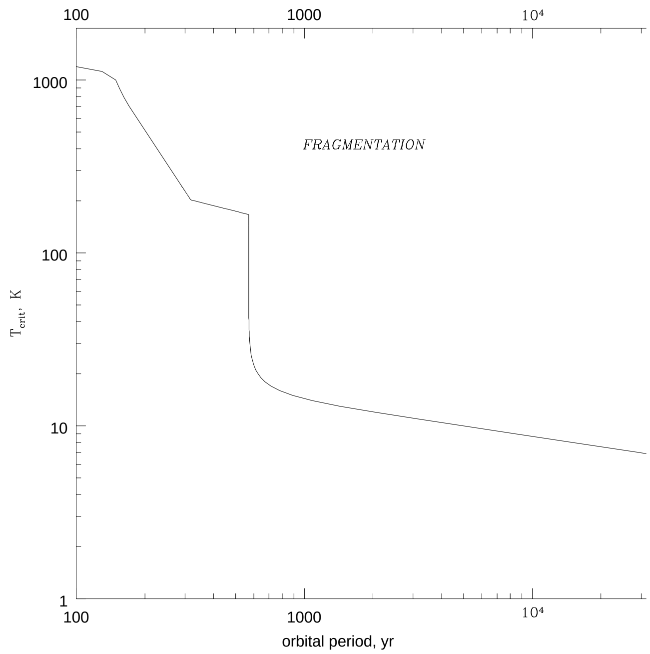

Figure 1. The temperature of the critically fragmenting disc as a function of the orbital period.

central source (about a thousand Schwarzschild radii from the supermassive black hole). We also, for the time being, neglect irradiation from the central source; this may be a good assumption if a disrupted molecular cloud forms a disc but the accretion onto the hole has not yet begun. As will be discussed below, irradiation is important for some regions in the AGN discs we are considering. However, as shown in the following subsection, inclusion of irradiation or other source of heating will only strengthen the case for formation of massive stars.

We assume, therefore, that the forming disc cools until it becomes self-gravitating; this happens when

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \simeq 1. \tag{1}$$

Here c_s is the isothermal speed of sound at the mid-plane of the disc, and Ω is the angular velocity of the disc. Numerical simulations show that once the disc becomes self-gravitating, turbulence and shocks develop; they transport angular momentum and provide heating which compensates the cooling of the disc (Gammie 2001). We thus assume, in agreement with the simulations, that when the disc exists, it is marginally self-gravitating, i.e. $Q = 1$. Then

$$c_s = \frac{\pi G \Sigma}{\Omega}, \tag{2}$$

and

$$T \sim 2 m_p c_s^2 / k_B = \frac{2 m_p}{k_B} \left(\frac{\pi G \Sigma}{\Omega} \right)^2. \tag{3}$$

Here T is the temperature in the mid-plane of the disc, m_p is the proton mass, and k_B is the Boltzmann constant.

The one-sided flux from the disc surface is given by the modified Stephan-Boltzmann law:

$$F = \sigma T_{\rm eff}^4, \tag{4}$$

where $T_{\rm eff}$ is the effective temperature. It is related to the mid-plane temperature by

$$T_{\rm eff}^4 \sim T^4 f(\tau) = f(\tau) \left(\frac{2 m_p}{k_B} \right)^4 \left(\frac{\pi G \Sigma}{\Omega} \right)^8 ., \tag{5}$$

where $\tau = \kappa \Sigma / 2$ is the optical depth of the disc; here $\kappa(T)$ is the opacity of the disc. The function $f(\tau) = \tau$ for an optically thin disc, and $f(\tau) = 1/\tau$ for an optically thick disc. We combine these two cases in our model by taking

$$f(\tau) = \frac{\tau}{\tau^2 + 1}. \tag{6}$$

We have used Eq. (3) in the last step of Eq. (5). The flux from the disc is powered by the accretion energy:

$$F = \frac{3}{8\pi} \Omega^2 \dot{M} = \frac{9}{8} \alpha \Omega c_s^2 \Sigma = \frac{9}{8} \alpha (\pi G)^2 \frac{\Sigma^3}{\Omega}. \tag{7}$$

Here α parametrizes viscous dissipation due to self-gravity (Gammie 2001); we have used Eq. (2) in the last step. Using Eqs (4), (7), and (5), we can express the viscosity parameter α as

$$\alpha = \frac{8\sigma}{9} \left(\frac{2 m_p}{k_B} \right)^4 (\pi G)^6 f(\tau) \frac{\Sigma^5}{\Omega^7}. \tag{8}$$

It is very important to emphasize that in this model α is only a function of Σ and Ω: the temperature in the mid-plane is determined by Eq. (3), and this temperature sets the opacity which in turn determines the optical depth $\tau = \kappa \Sigma / 2$. The opacity in the range of temperatures and densities of interest to us is set by light scattering off ice grains and, in some cases, by scattering off metal dust. The relevant regimes are worked out in the literature on protoplanetary discs; we use the analytical fit to the opacity (in $\rm cm^2/gm$) from the appendix of Bell and Lin, 1994.

$$\begin{array}{rll} \kappa = 0.0002 \times T_K^2 & \text{for} & T < 166\text{K}, \\ \kappa = 2 \times 10^{-16} T_K^{-7} & \text{for} & 166\text{K} < T < 202\text{K} \\ \kappa = 0.1 \sqrt{T_K} & \text{for} & T > 202\text{K}. \end{array} \tag{9}$$

In the first interval the opacity is due to ice grains; in the second interval the ice grains evaporate, and the opacity drops sharply with the temperature; in the third interval (the highest T) dust grains are the major source of the opacity.

From Eq. (8) we see that as Σ of the disc increases due to the merger-driven infall, the effective viscosity will reach $\alpha_{\rm crit} \sim 1$. At this stage the cooling time of the disc becomes comparable to the orbital period. Gammie's simulations show that in this case the turbulence induced by self-gravity is no longer able to keep the disc together, and the disc fragments. Gammie's simulations give $\alpha_{\rm crit} \simeq 0.3$; the numerical value we quote disagrees with Gammie's, but agrees with $\alpha_{\rm crit}$ quoted by Goodman (2003) since like Goodman we use isothermal speed of sound for α-prescription. Gammie's results, although obtained for razor-thin discs, justify the key assumption of our model: *the disc exists as a whole for* $\alpha < \alpha_{\rm crit}$, *and fragments once* $\alpha = \alpha_{\rm crit}$. Similar criterion for the disc fragmentation was already used in Shlosman and Begelman, 1987.

We shall refer to the mid-plane temperature and surface density of the marginally fragmenting disc with $\alpha = \alpha_{\rm crit}$ as the critical temperature $T_{\rm crit}$ and the critical surface density, $\Sigma_{\rm crit}$.

We now find the critical surface density and mid-plane temperature as a function of Ω. We use Eq. (3) to express Σ as a function of T and Ω, then substitute this function into Eqs. (6) and (8), and set $\alpha = \alpha_{\rm crit}$. After simple algebra, we obtain

$$\Omega^3 + p\Omega = q, \quad (10)$$

where

$$p = 2\left(\frac{\pi G}{\kappa(T_{\rm crit})}\right)^2 \frac{m_p}{k_B T_{\rm crit}},$$

$$q = \frac{32\sigma}{9\alpha_{\rm crit}}\frac{(\pi G)^2}{\kappa(T_{\rm crit})}(m_p/k_B)^2 T_{\rm crit}^2. \quad (11)$$

There is an analytical solution to Eq. (10):

$$\Omega = w - p/(3w), \quad (12)$$

where

$$w = \{q/2 + [(q/2)^2 + (p/3)^3]^{1/2}\}^{1/3}. \quad (13)$$

In Figure 1 we make a plot of $T_{\rm crit}$ as a function of the orbital period, for concreteness we set $\alpha_{\rm crit} = 0.3$. The critically self-gravitating disc is optically thin if the second term of the LHS of Eq. (10) is dominant, and optically thick otherwise. This can be expressed as a condition on the critical temperature: the disc is optically thin if

$$T_{\rm crit} < 12{\rm K}(\alpha_{\rm crit}/0.3)^{2/15}, \quad (14)$$

and optically thick for higher critical temperatures. The angular frequency above which the critically unfragmented disc becomes optically thick is

$$\Omega_{\rm transition} \simeq 16.3\times 10^{-11}{\rm sec}^{-1}. \quad (15)$$

We use Eqs. (2) and (3) to find the critical surface density $\Sigma_{\rm crit}$, which is plotted in Fig. 2, and the scaleheight $h_{\rm crit} = c_s/\Omega$ of a marginally fragmenting disc. The Toomre mass $\bar{M}_{\rm cl} = \Sigma_{\rm crit} h_{\rm crit}^2$ is the mass scale of the first clumps which form in the first stage of fragmentation. In Fig. 3, we plot the Toomre mass of the critically fragmenting disc as a function of the orbital period.

The value of $\bar{M}_{\rm cl}$ is not large enough for the initial clump to open a gap in the accretion disc. The newly-born clump will therefore accrete from the disc. The Bondi-Hoyle estimate of the accretion rate gives $\dot{M}_{\rm cl} \sim \Omega\bar{M}_{\rm cl}$, i.e. we expect the mass of the new clump to grow on the dynamical timescale until it becomes large enough to open a gap in the gas disc. The upper limit $\tilde{M}_{\rm cl}$ of this value is the mass which opens a gap in the original gas disc with $\Sigma = \Sigma_{\rm crit}$ just before it fragments:

$$\tilde{M}_{\rm cl} \simeq \bar{M}_{\rm cl}\left[12\pi(\alpha_{\rm crit}/0.3)\right]^{1/2}(r/h_{\rm crit})^{1/2}; \quad (16)$$

see Eq. (4) of Lin and Papaloizou (1986). Once the gas is depleted from the disc, we expect the initial distribution of the clump masses to be concentrated between $\bar{M}_{\rm cl}$ and $\tilde{M}_{\rm cl}$. The clump masses will evolve when clumps begin to merge with each other; this is addressed in section III.

2.0.0.1 Effect of irradiation and other sources of heating . So far in determining the structure of the self-gravitating disc, we have neglected external or internal heating of the disc. This is certainly a poor approximation in many cases. Irradiation from AGN or surrounding stars, or feedback from star formation inside the disc can be the dominant source of heating of the outer parts of AGN discs (eg, Shlosman and Begelman 1987, 1989). For example, irradiation from circumnuclear stars will keep the disc temperature at a few tens of Kelvin, which is higher than the critical temperature we obtained for a self-gravitating disc beyond 0.1pc away from $10^7 M_\odot$ black hole; see our discussion for the Galactic Center disc in section 4.

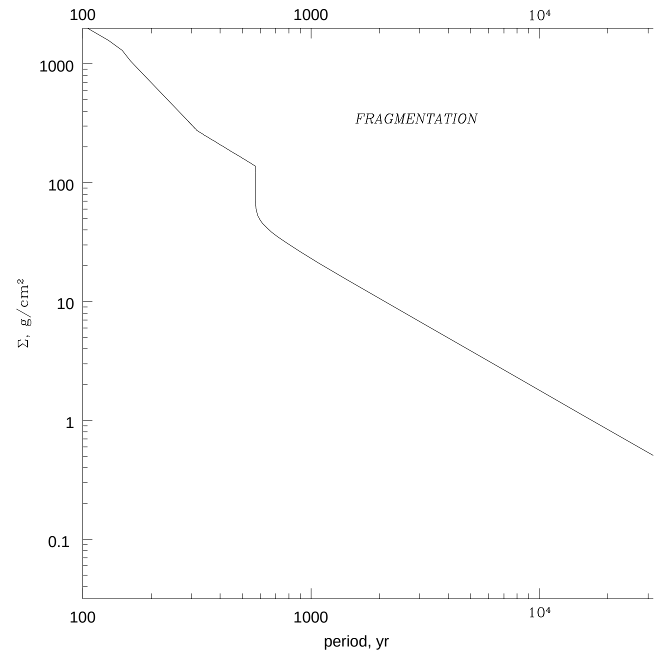

Figure 2. The surface density of the critically fragmenting disc as a function of the orbital period.

The extra heating will always work to increase the critical temperature at which the disc fragmentation occurs. Therefore, the values of the critical surface density $\Sigma_{\rm cr}$, scaleheight $h_{\rm cr}$, and the mass of the initial fragment $M_{\rm cl}$ obtained above should be treated as lower bounds of what might be expected around real AGNs or protostars. Higher values of these quantities would only strengthen main conclusions of this paper. LB have found that when the rate of accretion $\dot{M}$ is constant throughout the disc, the Toomre mass $\bar{M}_{\rm cl}$ is given by

$$\bar{M}_{\rm cl} = 1.8M_\odot\left(\frac{\alpha}{0.3}\right)^{-1}\times \frac{\dot{M}c^2}{L_{\rm edd}}\left(\frac{M}{3\times10^6M_\odot}\right)^{0.5}\left(\frac{r}{0.2{\rm pc}}\right)^{1.5}, \quad (17)$$

and that the gap-opening mass is

$$M_{\rm gap} = 62M_\odot\left(\frac{\alpha}{0.3}\right)^{-0.5}\times \frac{\dot{M}c^2}{L_{\rm edd}}\left(\frac{M}{3\times10^6M_\odot}\right)^{0.5}\left(\frac{r}{0.2{\rm pc}}\right)^{1.5}\left(\frac{r}{10h}\right)^{0.5}. \quad (18)$$

Here, M is the black-hole mass, and r and h are the radius and the scaleheight of the disc. Thus, even if the clumps do not merge with each other and stop their growth at the gap-opening mass, for high accretion rate the mass of the formed stars will be biased towards the high-mass end. In the next section we discuss the effects of clump mergers and the mass range of stars born after the disc fragments.

3 EVOLUTION OF THE FRAGMENTED DISC

Gammie's simulations show that once the disc fragments, the clumps merge and form significantly larger objects. In fact, his razor-thin shearing box turned into a single gas lump at the end of his simulation.

For merger to be possible, the clumps should not collapse into individual stars before they can coalesce with each other. Let's check if this is the case.

Consider a spherical non-rotating clump of radius R and mass $M_{\rm cl}$. First, assume that the clump is optically thin. The energy radiated from the clump per unit time is

$$W_{\rm cool} \sim \sigma T^4 R^2 \kappa(T)\Sigma \sim M_{\rm cl}\sigma T^4 \kappa(T). \quad (19)$$

This radiated power cannot exceed the clumps gravitational binding energy released in free-fall time, $G^{1.5}M^{2.5}R^{-2.5}$. Together with $\kappa(T) \propto T^2$ [since ice grains dominate opacity for the optically-thin marginally fragmenting disc–see Eq. (14)], this condition implies that

$$T < \tilde{T} = T_0 R^{-5/12}, \quad (20)$$

where T_0 is a constant for the collapsing optically thin clump. The temperature $\tilde{T}$ in Eq. (28) is less than the virial temperature, which scales as R^{-1}. Therefore, after the collapse commences, the clump is not virialized while it is optically thin. The temperature cannot be much smaller than $\tilde{T}$, since otherwise the cooling rate would be much smaller than the rate of release of the gravitational binding energy, and the gas would heat up by quasi-adiabatic compression. The inequality in Eq. (28) should be substituted by an approximate equality, and therefore we have during optically-thin collapse

$$T \propto R^{-5/12}. \quad (21)$$

The optical depth scales as

$$\tau \propto R^{-11/3}. \quad (22)$$

and hence rises sharply as the clump's radius decreases; as the clump shrinks it becomes optically thick†. It is possible to show that once the clump is optically thick, it virializes quickly with it's temperature $T \propto R^{-1}$. For $\kappa \propto T^2$ (ice grains), the cooling time of an optically thick clump scales with the clump radius as

$$t_{\rm cool} \propto R^{-3}. \quad (23)$$

The characteristic timescale for the clump to collide with another clump scales with the clump radius as

$$t_{\rm collision} \propto R^{-2}. \quad (24)$$

From Eqs. (23) and (24), we see that the collision rate decreases less steeply than the cooling rate as a function of the radius of an optically thick clump. Therefore, merger can be an efficient way of increasing the clump's mass.

This conclusion is no longer valid when the temperature of the clump becomes larger than $\sim$ 200K; then the opacity is dominated by metal dust with $\kappa \propto T^{1/2}$. In this case the cooling time scales as $t_{\rm cool} \propto R^{-1.5}$. The collision timescale increases faster than the cooling time as the clump shrinks, and naively one would expect that mergers may not be efficient in growing the clump masses.

However, we have neglected the rotational support within a clump. Each clump is initially rotating with angular frequency comparable to the clump's inverse dynamical timescale; for example, in a Keplerian disc each clump's initial angular velocity is $\sim \Omega/2$. Therefore each clump will shrink and collapse into a rotationally supported disc, and the size of this disc is comparable to the size of the original clump (this picture seems to be in agreement with Gammie's simulations). Thus rotational support generally slows down the collapse of an individual fragment and makes mergers between different fragments to be efficient.

Magnetic braking is one of the ways for the clump to lose its rotational support‡ (see, e.g., Spitzer 1978). One generally expects a horizontal magnetic field to be present in a differentially rotating disc due to the MRI (Balbus and Hawley, 1991). Ionization fraction in the disc is expected to be small, so the magnetic field is saturated at a sub-equipartition value $B = \beta B_{\rm eq}$, with $\beta << 1$. Horizontal magnetic field will couple inner and outer parts of the differentially rotating clump on the Alfven crossing timescale $t_{\rm alfven} \sim t_{\rm dynamical}/\beta$, and the collapse will proceed on this timescale as well.

What is the maximum mass that the clump can achieve? This issue has been analyzed for the similar situation of a protoplanetary core accreting from a disc of planetesimals (Rafikov 2001 and references therein). The growing clump cannot accrete more mass than is present in it's "feeding annulus". This gives the maximum "isolation" mass of a clump:

$$M_{\rm is} \sim \frac{(2\pi r^2 \Sigma_{\rm crit})^{3/2}}{M^{1/2}} = 2\pi\sqrt{2}\bar{M}_{\rm cl}(r/h_{\rm crit})^{3/2}, \quad (25)$$

where, as above, $\bar{M}_{\rm cl} = \Sigma_{\rm crit}h^2_{\rm crit}$ is the mass scale of the first clumps to form from a disc; see, e.g., Eq. (2) of Rafikov (2001). However, numerical work of Ida and Makino (1993) and analytical calculations of Rafikov (2001) indicate that the isolation mass may be hard to reach. The consider a massive body moving on a circular orbit through a disc of gravitationally interacting particles, and they find that when the mass of the body exceeds some critical value, an annular gap is opened in the particle disc around the body's orbit. We can idealize a disc consisting of fragments as a disc of particles of a typical fragment mass $M_{\rm fr}$. Once a growing clump opens a gap in a disc of gravitationally interacting fragments, the clump's growth may become quenched. This gap-opening mass of the clump $M_{\rm gap}$ is given by Eq. (25) of Rafikov (2001):

† The contraction of an optically thin clump may be complicated by sub-fragmentation, since the Jean's mass for such clump scales as $R^{7/8} \propto \tau^{-0.23}$. We suspect that in most cases the clump becomes optically thick before it sub-fragments, since the Jean's mass has a slow dependence on the optical depth. However, only detailed numerical simulations can resolve these issues.

‡ Another way is via collisions with other clumps.

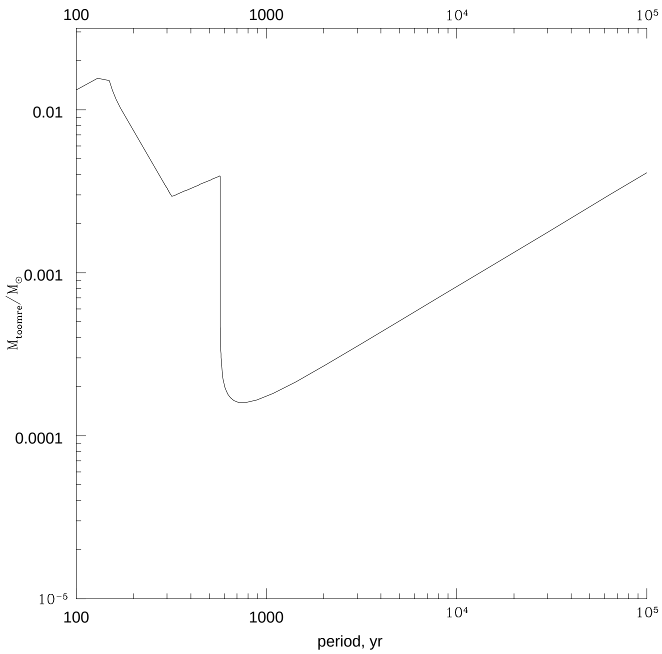

Figure 3. The Toomre mass of the critically fragmenting disc as a function of the orbital period.

$$\frac{M_{\rm gap}}{M_{\rm is}} = \frac{I}{2^{7/6}\pi^{1/2}}\left(\frac{M_{\rm fr}}{\Sigma r^2}\right)^{1/3}\left(\frac{M}{\Sigma r^2}\right)^{1/2}. \quad (26)$$

We use the numerical factor $2^{-7/6}\pi^{-1/2}I = 1.5$ appropriate for thin discs. By taking $Q = 1$ we get

$$M_{\rm gap} \simeq 14 M_{\rm fr}(M_{\rm fr}/\bar{M}_{\rm cl})^{1/3}(r/h_{\rm crit}). \quad (27)$$

In Figure 4 the masses $M_{\rm is}$ and $M_{\rm gap}$ are plotted as a function of radius for a $3\times10^6 M_\odot$ black hole; when we calculate $M_{\rm gap}$ we conservatively set $M_{\rm fr} = \bar{M}_{\rm cl}$ and not to the larger value $\tilde{M}_{\rm cl}$. It is likely that the most massive clumps will reach $M_{\rm gap}$, but it will be more difficult to form a clump with the mass $M_{\rm is}$.

From Fig. 4 we see that the most massive clumps can reach tens hundreds of solar masses. The expected e-folding time for the clump growth is comparable to the orbital period, and thus these high masses will be reached within ~ 20 orbital periods. The maximum mass would be even larger if we included the heating of the disc by external irradiation or internal starburst. It is plausible that these very massive clumps will form massive stars; the masses of the stars may be comparable to the masses of the original clumps; see McKee and Tan (2002) and references therein.

Goodman and Tan (2004) have argued that a protostar accreting from the accretion disc will reach the isolation mass; see also Nayakshin and Cuadra (2005) and Nayakshin (2005). This conclusion was criticized by Milosavljevic and Loeb (ML, 2004), who have shown that the mini-accretion disc around the star itself will be unstable to fragmentation. Our picture of merging clumps forming a massive protostellar core is not susceptible to the ML's criticism.

4 YOUNG STARS NEAR SGRA*

4.1 Fragmenting disc

So far we have mostly considered an idealized situation where the disc is self-luminous and not irradiated externally. However, a gaseous disc in the Galactic Center (when there is one) is heated from outside by the light from bright stars in the cusp (mostly UV), and re-radiates this in the infrared. If the disc is optically thick to its thermal radiation, then the temperature T of the disc is uniform in vertical direction and is determined by the Stefan-Boltzmann law:

$$\sigma T^4 \sim \frac{L_{\rm irradiation}}{\pi R^2}. \quad (28)$$

where R is the disc radius. If the disc is optically thin to its thermal radiation, then determining the vertical temperature profile of the disc requires more care. The superheated dust layer forms on its skin and the interior is heated by the infrared light emitted by this layer (Chiang and Goldreich, 1997). Two cases can be distinguished: first, in which the disc interior is thin to its own thermal radiation but thick to the one from superheated dust, and second, in which the disc is thin to both its own radiation and that of the dust. In both cases, however, the mid-plane temperature will be slightly, but not much, greater then the estimate in Eq. (28) (i.e., by a factor of order $\tau^{-1/4}$ in the first case and $(\epsilon_s/\epsilon_i)^{1/4}$ in the second case; here τ is the disc's optical depth to its thermal radiation and $\epsilon_{s,i}$ are the dust emissivities in the superheated layer and mid-plane, respectively). We will see that the actual optical depth of the disc to its thermal radiation will be of order 1.

The current total luminosity of stars is estimated to be $L_0 = 2\times10^7 L_\odot$ (Paumard et al., 2005). Lets assume that a fraction β of the stellar light is intercepted by the disc (this fraction is of order 1 since the disc is optically thick for the UV). From Eq. (28), the disc temperature is[§]

$$T \simeq 84\beta^{1/4}(L/L_0)^{1/4}(0.5{\rm pc}/R)^{1/2}{\rm K}. \quad (29)$$

and the speed of sound is

$$c_s \simeq 4.5\times10^4\beta^{1/8}(L/L_0)^{1/8}(0.5{\rm pc}/R)^{1/4}{\rm cm/sec} \quad (30)$$

From Figure 1 we see that for orbital periods greater than 600 years (this corresponds to $r > 0.06$pc for SgrA*) the temperature of self-luminous disc is smaller than that of the gaseous disc in the Galactic Center, and thus the disc is supported by external irradiation prior to fragmentation. At the fragmentation boundary, the Toomre Q parameter equals 1:

$$Q = \frac{c_s\Omega}{\pi G\Sigma} = 1 \quad (31)$$

[§] The disc will be also heated by the stars from the central star cluster when they collide with the gas (Syer et al., 1981). The rate of heating per area can be estimated as $\sim \Sigma r_*^2 n\sigma^3 \simeq 5L_\odot/{\rm pc}^2$, where Σ is the disc column density, r_* is the typical stellar radius, and n and σ are the density of stars and the velocity dispersion of the cluster, respectively. In the numerical estimate we have taken the values appropriate for the Galactic Center environment: $r = r_\odot$, $\Sigma = 10{\rm g/cm}^2$, $n = 10^6{\rm pc}^{-3}$, and $\sigma = 1000$km/s. Clearly, the power input from this process is negligibly small compared to that from the external irradiation.

and thus from Eq. (30),

$$\Sigma = 2.7 \times (0.5\mathrm{pc}/r)^{-3/2} (\beta L/L_0)^{1/8} (0.5\mathrm{pc}/R)^{1/4} \mathrm{g/cm}^2 \quad (32)$$

Thus the *mass* of the fragmenting disc is

$$M_{\rm disc} = 4\pi (R^2 * \Sigma(R) - R_{\rm in}^2 \Sigma(R_{\rm in}))$$
$$= 2.5 \times 10^4 [(R/0.5\mathrm{pc})^{1/2} - (R_{\rm in}/0.06\mathrm{pc})^{1/2}] \times \quad (33)$$
$$(\alpha L/L_0)^{1/8} (0.5\mathrm{pc}/R)^{1/4} M_\odot, \quad (34)$$

where $R_{\rm in}$ is the inner radius of the disc. I have taken this inner radius to be $R_{\rm in} = 0.06$pc, since inside this radius the critical column density necessary to achieve fragmentation is much larger than outside, see Fig. 2. The value of $R_{\rm in}$ agrees well with the observed inner edge of the clockwise stellar disc, which is 0.03pc in projection on the sky (Paumard et al., 2005). The disc mass obtained in the Equation above appears to be twice as large as the value estimated from observations by Paumard et al. (2005); see also Nayakshin et al. (2005). The difference can be explained if either (a) the stellar luminosity was smaller by ~ 300 than the current one during the disc formation; this is unlikely since even though most of the current luminosity is due to the recently-formed stars, there is substantial UV flux from post-AGB stars in older populations and the center of the galaxy has significant soft-x-ray sources which should predate the most recent starburst, (b) the starburst occupied the outer half of the disc, or (c) only half of the gas was converted into stars, and the rest left the GC as wind.

Equation (32) makes a prediction that $\Sigma \propto r^{-1.5}$ during fragmentation. But what about the number density of formed stars? If the typical stellar mass scales with the isolation mass of the disc (Goodman and Tan 2004, section 3 of this paper), then

$$M_{\rm isolation} \propto (\Sigma r^2)^{3/2} \propto r^{0.75}, \quad (35)$$

and the number of stars per unit are scales as

$$dN/d(Area) \propto r^{-2.25}. \quad (36)$$

This is in good agreement with the data in Paumard et al. (2005), who find $dN/d(Area) \propto r^{-2}$

4.2 S-stars

It is not easy to visualize how eccentric orbits orbits of the S-stars could be consistent with their birth in a disc, and so far the ideas for their origin have invoked stellar dynamics. Recently, two interesting suggestions have been put forward. Alexander and Livio (2004) have proposed that the S-stars got captured by an exchange interaction with stellar-mass black holes in the SgrA* cusp; however this scenario seems to be disfavored by current observations (Paumard et al, 2005). Gould and Quillen (2003) have argued that in the past the S-stars were members of binaries. When the binary passes close to SgrA*, one of its members gets ejected and the other remains bound to SgrA* on an eccentric orbit. However, (a) it is unclear how a young binary would get onto a plunging orbit during its lifetime, and (b) some of the S-stars' eccentricities are not high; for example S1 and S13 have eccentricities of around 0.4 (Eisenhauer et al., 2005). It is certainly impossible to capture stars on orbits with such low eccentricity by disruption of stellar binaries, since the typical eccentricity scales as $(1-e) \sim (m/M)^{1/3}$

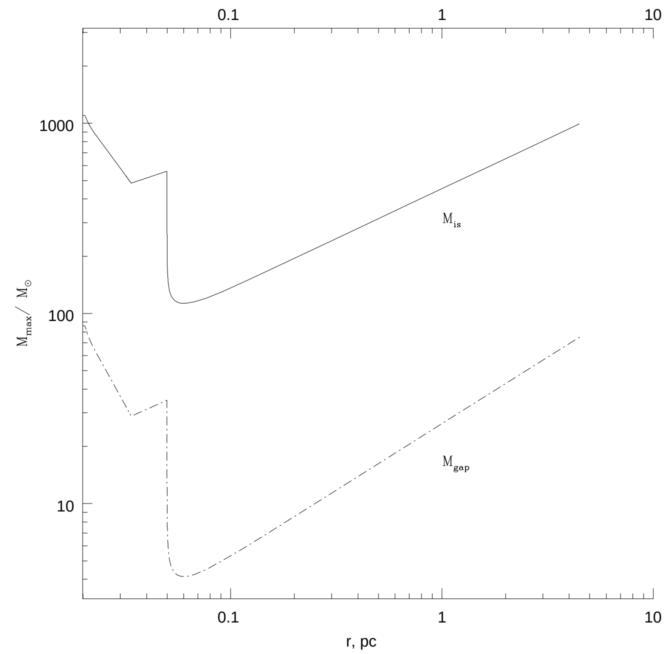

Figure 4. The isolation and gap opening masses plotted as a function of radius for the critically fragmenting disc without external sources of heating. The black hole mass is taken to be $3 \times 10^6 M_\odot$.

where m and M is the mass of the binary and SgrA*, respectively. Therefore, one has to appeal to a fast relaxation twice: the first time, to get the binary on a plunging orbit when it is far from SgrA*, and the second time, to get the star's eccentricity relaxed to a lower value when it is close to SgrA*.

Rauch and Tremaine (1996) have identified a fast relaxation process near supermassive black holes, and called it the "resonant relaxation" (RR). Here we argue that it is potentially important for the S-stars; this argument has also been made independently by Hopman and Alexander (2006)¶ The characteristic timescale for the eccentricity change due to RR is given by

$$t_{RR} \simeq (0.5/\beta_s)^2 \left(\frac{M}{4 \times 10^6 M_\odot}\right) \left(\frac{16 M_\odot}{m_s}\right) (P_{\rm orb}/\mathrm{yr}) \times 10^6 \mathrm{yr}, (37)$$

where m_s is the typical mass of a star in the cusp [which is thought to be dominated by stellar-mass black holes in the central 0.1pc, see Miralda-Escude and Gould (2000)], $P_{\rm orb}$ is the orbital period of the star, and β_s is the numerical coefficient, found to be approximately 0.5 by Rauch and Tremaine. Equation (37) is valid so long as relativistic precession is longer than the Newtonian precession due to the cusp. Otherwise, the expression for t_{RR} has to be multiplied by the ratio of the two precession frequencies. The longest measured period of the S-star is that of S1,

¶ We acknowledge Scott Tremaine for attracting our attention to the RR as a potentially important relaxation mechanism for the S-star relaxation, in 2004.

which has $P_{\rm orb} = 94$yr (Eisenhauer et al., 2005, see also Ghez et al., 2005). Thus the longest RR timescale for the S-star is of order 100Myr, well within a possible lifetime of the main sequence B stars. The RR relaxation timescales are shorter for all the other stars, the shortest one being for S2: $t_{RR} \simeq 40$Myr. Given that we do not know the typical mass of a stellar-mass black hole in the cusp[‖], we conclude that **RR is a viable mechanism for relaxation of orbits of all of the S-stars within their lifetime**.

However, there is no mechanism known to us which would place the young stars on plunging orbits when they are further out from the SgrA*, as required in both Gould and Quillen (2003) and Alexander and Livio (2004) scenarios. We therefore suggest that the S-stars were born during the previous starburst or several starbursts, and then migrated towards smaller radii through their parent discs. This possibility was already pointed out in LB; it is known as type-I migration in planetary dynamics. The migration velocity through the disc is given by

$$v_{\rm in} \sim 2\beta \frac{G^2 m\Sigma h}{c_s^3 r}; \tag{38}$$

cf. Eqs. (9), (B4) and (B5) of Rafikov (2002), and Ward (1986). Here β is a numerical factor of order 5 for $Q \gg 1$, and can be significantly larger for $Q \sim 1$. From Eq. (30) we see that the migration timescale for an S-star embedded in the disc at 0.1pc is of order 10^5yr, i.e. much shorter than the star's lifetime. Complication arises if the star opens a gap in the disc. However, in this case a fast runaway migration is possible due to co-rotation torques acting on the star; the speed of the runaway migration is comparable to that of type-I migration (Artymowicz 2003, Masset and Papaloizou 2003).

5 MERGER OF THE CENTRAL BLACK HOLE AND THE DISC-BORN BLACK HOLE.

Stars with masses of a few tens of solar masses will produce black holes as the end product of their rapid ($\sim 10^6$yr) evolution; the characteristic mass of these black holes is believed to be around $10M_\odot$. A recent work by Mirabel and Rodrigues (2003) shows that the black holes whose progenitors have masses $> 40M_\odot$ do not receive a velocity kick at their birth. Thus, significant fraction of the disc-born black holes will remain embedded in the disc.

It is likely that the newly-born black hole in the disc inspirals towards the central black hole. We imagine that the stellar-mass black hole is embedded into a massive accretion disc which forms due to continuing infall of gas from the galactic bulge, after the black hole is born. If the black hole opens a gap in the disc, it will move towards the central black hole together with the disc (type-II migration; Gould and Rix 2000, and Armitage and Natarajan 2001). The timescale for such inspiral is the accretion timescale,

$$t_{\rm inspiral} \sim 10^6 {\rm yr} \frac{M_7^{-1/2}}{\alpha_{0.1}} \left(\frac{0.1{\rm pc}}{r}\right)^{-3/2} \left(\frac{r}{30h}\right)^2. \tag{39}$$

‖ Top-heavy IMF may imply top-heavy black holes!

If on the other hand, the black hole is not massive enough to open the gap, it will migrate inwards by exciting density waves in the disc (type-I migration, see previous section). The speed of this inward drift is given by Eq. (38). For a self-gravitating disc with $Q \sim 1$, we find

$$t_{\rm inspiral} \sim 10^7 {\rm yr} \frac{5}{\beta} \frac{30h}{r} \frac{10M_\odot}{M_{\rm bh}} \left(\frac{r}{0.1{\rm pc}}\right)^{1.5} M_7^{0.5}. \tag{40}$$

What determines whether the gap is open or not is whether a stellar-mass black hole has time to accrete a few thousand solar masses of gas, which would put it above the gap-opening threshold for the typical disc parameters. The Eddington-limited accretion occurs on a timescale of $\sim 10^8$yr, much longer than the characteristic type-I inspiral time. Thus, in this case the black holes don't accrete much on their way in. On the other hand, the Bondi-Hoyle formula predicts the mass e-folding timescale of a few hundred years. Thus, if the black hole is allowed to accrete at the Bondi-Hoyle rate, its mass increases rapidly until it opens a gap in the disc. Then, the inspiral proceeds via type-II migration.

From Eqs. (39) and (40) we see that the embedded black hole experiencing type-I or type-II migration would merge with the central black hole on the timescale 10^6—10^7 years, shorter than the typical timescale of an AGN activity. Thus it is plausible that the daughter disc-born black hole is brought towards the parent central black hole; the mass of the daughter black hole might grow significantly on the way in. Gravitational radiation will eventually become the dominant mechanism driving the inspiral, and the final merger will produce copious amount of gravitational waves. In the next subsection we show that these waves are detectable by LISA for a broad range of the black hole masses and the disc accretion rate.

5.1 Influence of the accretion disc on the inspiral signal as seen by LISA

It is realistic to expect that LISA would follow the last year of the inspiral of the disc-born black hole into the central black hole. Generally, one must develop a set of templates which densely span the parameter space of possible inspiral signals. In order for the final inspiral to be detectable, one of the templates must follow the signal with the phase-shift between the two not exceeding a fraction of a cycle. Therefore, if the drag from the disc will alter the waveform by a fraction of a cycle over the signal integration time (e.g., 1 year), detection of the signal with high signal-to-noise ratio will become problematic. Below we address the influence of the accretion disc on the final inpiral waveform.

The issue of gas-drag influence on the LISA signal was first addressed by Narayan (2000); see also Chakrabarti (1996). Narayan's analysis is directly applicable to low-luminosity non-radiative quasi-spherical accretion flows, which might exist around supermassive black holes when the accretion rate is < 0.01 of the Eddington limit. Narayan concluded that non-radiative flows will not have any observable influence on the gravitational-wave signals seen by LISA. Below we extend Narayan's analysis to the case of radiative disc-like flows with high accretion rate, which are likely to be present in high-luminosity AGNs.

Consider a non-rotating central black hole of mass $M_6 \times 10^6 M_\odot$, accreting at a significant fraction $\dot{m}$ of the Eddington rate, $\dot{M} = \dot{m}\dot{M}_{\rm edd}$. The accretion disc in the region of interest ($< 10R_S$, where R_s is the Schwartzschild radius of the central black hole) is radiation-pressure dominated, and the opacity $\kappa = 0.4{\rm cm}^2/{\rm g}$ is due to the Thompson scattering. By following the standard thin-disc theory** (Shakura and Sunyaev, 1973), we get

$$\Sigma(r) = \frac{64\pi c^2}{27\alpha\Omega\kappa^2\dot{M}} \sim 4{\rm g/cm}^2\frac{\epsilon}{\alpha\dot{m}}\left(\frac{r}{r_s}\right)^{3/2}, \quad (41)$$

where ϵ is the efficiency with which the accreted mass converts into radiation, and r_s is the Schwartzschild radius of the central black hole; and

$$c_s = \frac{3\dot{M}\Omega\kappa}{8\pi c}. \quad (42)$$

The disc black hole in orbit around the central black hole excites density waves in the disc (Goldreich and Tremaine, 1980); these waves carry angular momentum flux

$$F_0 \sim (GM_{\rm bh})^2\frac{\Sigma r\Omega}{c_s^3}; \quad (43)$$

here, as before, $M_{\rm bh}$ is the mass of the orbiting disc black hole. Ward (1987) has argued that the torque acting on the orbiting body is $dL_{\rm dw}/dt \sim (h/r)F_0$. We can compute the characteristic timescale for the orbit evolution due to the density-waves torque:

$$t_{\rm dw} = \frac{L}{dL_{\rm dw}/dt} = \frac{1}{\Omega}\frac{M}{M_{\rm bh}}\frac{M}{\Sigma r^2}\left(\frac{h}{r}\right)^2. \quad (44)$$

One must compare $t_{\rm dw}$ to the timescale $t_{\rm gw}$ of orbital evolution due to gravitational-radiation-reaction torque:

$$t_{\rm gw} = 8t_{\rm m} = \frac{5}{8}\frac{cr_s^2}{GM_{\rm bh}}\left(\frac{r}{r_s}\right)^4. \quad (45)$$

Here, t_m is the time left before the disc and central holes merge, i.e. the integration time for the LISA signal. Optimistically we could expect to follow the LISA signal to 0.1 of a cycle. Therefore, if $q = 10n_{\rm m}t_{\rm gw}/t_{\rm dw}$ is less than unity, the disc drag does not impact detection of the final inspiral; cf. Eq. (16) of Narayan (2000). Here $n_{\rm m} = \Omega t_{\rm gw}/(5\pi)$ is the number of cycles the disc black hole will make before merging with the central black hole. Using Eqs. (41), (42), (44), and (45), we get

$$q \simeq 2\times 10^{-7}\frac{\epsilon_{0.1}^3}{\dot{m}^3\alpha_{0.1}}\frac{(M_{\rm bh}/10M_\odot)^{13/8}}{M_6^{13/4}}t_{m,yr}^{21/8}, \quad (46)$$

where $\epsilon = 0.1\epsilon_{0.1}$, $\alpha = 0.1\alpha_{0.1}$, and $t_m = 1{\rm yr} * t_{m,yr}$. We see that for a large range of parameters $q < 1$, and the disc drag does not influence the inpiral signal. However, note that q is a very sensitive function of $\dot{m}$ and $M_{\rm bh}$. For instance, the disc will influence significantly an inspiraling $100M_\odot$ black hole when the accretion rate is down to a few percent of the Eddington limit. One then needs to reduce the influence of the disc on the LISA signal by choosing to observe the smaller portion of the final inspiral, i.e. by choosing a smaller integration time t_m.

** The disc is no longer thin close to the central black hole, however our estimates of the disc structure should be correct to an order of magnitude.

There is another important source of drag experienced by the inspiraling black hole; it was first analyzed by Chakrabarti (1993, 1996). Generally, there is a radial pressure gradient in an accretion disc; this pressure gradient makes the azimuthal velocity of the disc gas slightly different from a velocity of the test particle on a circular orbit at the same radius. Therefore the inspiraling black hole will experience a head wind from a gas in the accretion disc; by accreting gas from the disc the black hole will experience the braking torque which will make it lose it's specific angular momentum. This torque is given by

$$\tau_{\rm wind} = \dot{M}_{\rm bh}\Delta v r, \quad (47)$$

where $\dot{M}_{\rm bh}$ is rate of accretion onto the inspiraling black hole from the disc, and $\Delta v \sim c_s^2/v_o$ is the speed of the headwind experienced by the black hole moving with the orbital speed v_o. We assume that the inspiraling hole accretes with the Bondi-Hoyle rate,

$$\dot{M}_{\rm bh} \simeq \pi\rho(GM_{\rm bh})^2/c_s^3, \quad (48)$$

where $\rho = \Sigma/h = \Sigma\Omega/c_s$ is the density of the ambient disc gas. By using the last equation in Eq. (47), we obtain the expression for the torque $\tau_{\rm wind}$, which turns out to be the same as the torque from the density waves, up to the numerical factor between 1 and 10. We see therefore that inclusion of Chakrabarti's "accretion" torque is important for the detailed analysis, but does not qualitatively change our conclusions.

Can the orbiting black hole open a gap in the disc during it's final inspiral? In order to overcome the viscous stresses which oppose opening the gap, the mass of the orbiting black hole should be greater than the threshold value given by

$$M_{\rm bhgap} \simeq \sqrt{40\alpha}(h/r)^{2.5}M, \quad (49)$$

see Eqs. (4) of Lin and Papaloizou (1986). The radius $r_{\rm in}$ from which the inspiral begins is given by

$$r_{\rm in} = 4(M_{\rm bh}/10M_\odot)^{1/4}M_6^{-1/2}t_{m,yr}^{1/4}r_s, \quad (50)$$

and the scaleheight of the disc is radius-independent in the radiation-dominated inner region (this is true only if you treat the central black hole as a Newtonian object):

$$h = \frac{3\dot{m}}{4\epsilon}r_s. \quad (51)$$

The gap will be open in the disc, therefore, if the mass of the inspiraling black hole exceeds

$$m_{\rm gap} \sim \dot{m}^{20/13}M_6^{-2/13}10^4M_\odot. \quad (52)$$

During the final year of merger of the $10M_\odot$ disc-born black hole and the $10^6M_{\rm odot}$ central black hole, the gap will be open if the accretion rate onto the central black hole is about five percent of the Eddington limit. Such gap-opening merger can result in sudden changes in the AGN luminosity and produce an optical counterpart to the gravitational-wave signal.

5.2 The merger event rate as seen by LISA

The details of black-hole formation and evolution in the disc are uncertain, and it seems impossible to make a reliable estimate of an event rate for LISA from this channel of black-hole mergers. Nonetheless, the expected top-heavy IMF gives us reason for optimism. It is worth working through a simple example to show that the merger of disc-born holes with the central hole may be an important source for LISA.

Studies of integrated light coming from Galactic Nuclei show that the supermassive black holes acquire significant part and perhaps almost all of their mass via accretion of gas; see, e.g., Yu and Tremaine (2002) and references therein. Assume, for the sake of our example, that a mass fraction η of this gas is converted into $100M_\odot$ black holes on the way in, and that all of these black holes eventually merge with the central black hole. LISA can detect such mergers to $z = 1$ if the central black hole is between 10^5 and 10^7 solar masses (one needs to assume that the central black hole is rapidly rotating at the high-mass end of this range). The mass density of such black holes in the local universe is $\sim 10^5 M_\odot/\mathrm{Mpc}^3$ (Salucci et. al. 1999). We can estimate, using Figure 3 of Pei et. al. (1995), that about 10 percent of integrated radiation from Galactic Nuclei comes from redshifts accessible to LISA, $z < 1$. This implies that supermassive black holes acquired 10 percent of their mass at $z < 1$. We shall therefore assume[††] that 10 percent of the mass of the black holes in LISA mass range was accreted at $z < 1$. This implies that there were $\sim 100\eta = \eta_{0.01}$ mergers per mpc^3 which are potentially detectable by LISA. When we multiply this by volume out to $z = 1$ and divide it by the Hubble time, we get an estimate of the LISA event rate from such mergers,

$$dN/dt_{\mathrm{toymodel}} \sim 10\eta_{0.01}/\mathrm{yr}. \tag{53}$$

The estimate above should be treated as an illustration of importance for our channel of the mergers, rather than as a concrete prediction for LISA.

Currently, it is not known how to compute well the gravitational waveform produced by an inspiral with an arbitrary eccentricity and inclination relative to the central black hole. This might pose a great challenge to the LISA data analysis. Indeed, in the currently popular astrophysical scenario, the stellar-mass black hole gets captured on a highly eccentric orbit with arbitrary inclination relative to the central black hole (Sigurdsson and Rees, 1997). By contrast, in our scenario the inspiral occurs in the equatorial plane, and the signal is well understood (Hughes 2001 and references therein). Circular inspiral is also expected in the binary-disruption scenario of Miller et al. 2005; however there the inspiral is not expected to be preferentially aligned with the equatorial plane. The template for detection of a circular inspiral is readily available, and the merger channel we consider has a clear observational signature which distinguishes it from other channels. It is an open question whether the inspiraling hole can acquire high eccentricity by interacting with the disc or with other orbiting masses (Goldreich and Sari 2003, Chiang et. al. 2002); it seems likely that at least in some cases interaction with the disc will act to circularize the orbit. Finn and Thorne (2000) have performed a detailed census of parameter space for circular-orbit equatorial inspirals as seen by LISA.

[††] This assumption has an extra caveat for the low-luminosity AGNs, significant fraction of which may be powered by tidal disruption of stars (Eracleous et al. 1995, Milosavljevic et al. 2006). We note that in one low-luminosity AGN, in the nucleus of Circinus galaxy, there is a firm evidence of an extended accretion disc (Greenhill et al. 2003).

6 CONCLUSIONS

In this paper, we rely on numerical simulations by Gammie (2001) to develop a formalism for self-gravitating thin discs which are gaining mass by continuous infall. We present a way to calculate the critical temperature, surface density, and scaleheigh of the disc just prior to fragmentation. Our formalism naturally includes both optically thin and optically thick discs.

We then speculate on the outcome of the nonlinear physical processes which follow fragmentation: accretion and merger of smaller fragments into bigger ones. We find an upper bound on the mass of final, merged fragments, and we give a plausibility argument that some fragments will indeed reach this upper bound. We thus predict that very massive stars of tens or even hundreds of solar masses will be produced in self-gravitating discs around supermassive black holes. This picture explains well recent observations of the stellar discs in the Galactic Center, and we clarify how the peculiar orbits of the S-stars may be generated by a combination of inward migration through the gaseous disc and Resonant Relaxation in the central arc-second.

The end product of fast evolution of the massive stars will be stellar-mass black holes. We make an argument that in the case when accretion is prolonged, the disc-born black holes in AGNs find a way to merge with central black holes. We consider a purely toy-model example of what the rate of such mergers might be, as seen by LISA; we illustrate this rate might be high enough to be interesting for future gravitational-wave (GW) observations. The GW signal from this merger channel is distinct from that of other channels, and can be readily modeled using our current theoretical understanding of the final stages of the inspiral driven by gravitational-radiation reaction. We show that for a broad range of accretion rates and black-hole masses the drag from accretion disc will not be large enough to pollute the signal and make inspiral template invalid. In some cases, the inspiraling hole will open a gap in the accretion disc close to the central black hole, thus producing a possible optical counterpart for the gravitational-wave burst generated by the merger.

7 ACKNOWLEDGMENTS

I have greatly benefited from discussions with Chris Matzner and Scott Tremaine. This paper is largely based on my earlier preprint (Levin, 2003). I thank Norm Murray and Clovis Hopman for encouraging me to publish it.

References

Alexander, T., & Livio, M. 2004, ApJ, 606, L21
Armitage, P. J., & Natarajan, P. 2002, ApJ, 567, L9
Artymowicz, P., 2003, Dynamics of Gaseous Discs with Planets, ASP Conference Series, v. 324

Artymowicz, P., Lin, D. N. C., & Wampler, J. 1993, ApJ, 409, 592
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Bate, M. R., Bonnel, I. A, & Bromm, V. 2002, MNRAS, 332, 65
Bell, K. R., & Lin, D. N. C. 1994, ApJ, 427, 987
Begelman, M. C. 2002, ApJ, 568, L97-100
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton University Press)
Brandt, W. N., Podsiadlowski, Ph., & Sigurdsson, S. 1995, MNRAS, 277, L35
Chakrabarti, S. K 1993, ApJ, 411, 610
Chakrabarti, S. K 1996, Phys. Rev. D, 53, 2901
Chiang, E. I., & Goldreich, P. 1997, ApJ, 490, 368
Chiang, E. I., Fisher, D., & Thommes, E. 2002, ApJ, 564, L105
Collin, S., & Zahn, J.-P. 1999, A&A, 344, 433
Eracleous, M., Livio, M., & Binette, L. 1995, ApJ, 445, L1
Eisenhauer, F., et al. 2005, ApJ, 628, 246
Finn, L. S., & Thorne, K. S. 2000, Phys. Rev. D, 62, 124021
Gammie, C. F. 2001, ApJ, 553, 174
Genzel, R., et. al. 2000, MNRAS, 317, 348
Genzel, R., et. al. 2003, ApJ, 594, 812
Gerhard, O. 2001, ApJ, 546, L39
Ghez, A. M., et al. 2005, ApJ, 620, 744
Greenhill, L.J, et al. 2003, ApJ, 590, 162
Goldreich, P., & Sari, R. 2003, ApJ, 585, 1024
Goldreich, P., & Tremaine, S. 1980, 241, 425
Goodman, J. 2003, MNRAS, 339, 937
Goodman, J., & Tan, J. C., 2004, ApJ, 608, 108
Gould, A., & Rix, H.-W. 2000, ApJ, 532, L29
Gould, A., & Quillen, A. C. 2003, ApJ, 592, 935
Hansen, B., & Milosavljevic, M. 2003, ApJ, 593, L77
Hopman, C., & Alexander, T. 2006, submitted to ApJ Letters, astro-ph/0603324
Hughes, S. A. 2001, Class. Quant. Grav., 18, 4067, and references herein
Ida, S.,& Makino, J., Icarus, 106, 210
Kolykhalov, P. I., & Sunyaev, R. A. 1980, SvAL, 6, 357
Kumar, P. 1999, ApJ, 519, 599
Levin, Y. 2003, astro-ph/0307084
Levin, Y., & Beloborodov, A. M. 2003, ApJ, 590, 33L
Lin, D. N. C., & Papaloizou, J. 1986, ApJ, 309, 846
Lin, D. N. C., & Pringle, J. E. 1987, MNRAS, 225, 607
Mayer, L., et al. 2002, Science, 298, 1756
Masset, M. F., & Papaloizou, J. C. P. 2003, ApJ, 588, 494
McKee, C. F., & Tan, J. C. 2002, Nature, 416, 59
Miller, M. C., et al. 2005, ApJ, 631, L117
Milosavljevic, M., & Loeb, A. 2004, ApJ, 604, L45
Milosavljevic, M., Merritt, D., & Ho, L. C. 2006, astro-ph/0602289
Mirabel, F., & Rodrigues, I. 2003, Science, 300, 1119
Miralda-Escude, J., & Gould, A. 2000, ApJ, 545, 847
Morris, M., Ghez, A. M., & Becklin, E. E. 1989, Advances in Space Research, 23, 959
Narayan, R. 2000, ApJ, 536, 663
Nayakshin, S., & Sunyaev, R. 2005, MNRAS, 364, 23
Nayakshin, S., & Cuadra, J. 2005, A&A, 437, 437
Nayakshin, S., et al. 2006, MNRAS, 346, 1410
Nayakshin, S. 2005, submitted to MNRAS, astro-ph/0512255
Paczysnki, B. 1978, AcA, 28, 91
Paumard, T., et al. 2006, to appear in ApJ (astro-ph/0601268)
Pei, Y. 1995, ApJ, 438, 623
Portegies Zwart, S. F., McMillan, S., & Gerhard, O. 2003, ApJ, 593, 352
Rafikov, R. R. 2001, AJ, 122, 2713
Rafikov, R. R. 2002, ApJ, 572, 566
Rice, W. K. M, et. al. 2003, MNRAS, 339, 1025
Salucci, P., Szuszkiewicz, E., Monaco, P.,& Danese, L. 1999, MNRAS, 307, 637
Sanders, R. H. 1998, MNRAS, 294, 35
Shakura, N. I, & Sunyaev, R. A. 1973, A&A, 24, 337
Shlosman, I., & Begelman, M. 1987, Nature, 329, 810
Shlosman, I., & Begelman, M. 1989, ApJ, 341, 685
Sigurdsson, S., & Rees, M. J. 1997, MNRAS, 284, 318
Sirko, E., & Goodman, J. 2002, MNRAS, 341, 501
Syer, D., Clarke, C.J., & Rees, M.J. 1991, MNRAS, 250, 505
Tan, J. C., & Blackman, E. 2005, MNRAS, 362, 983
Thompson, T. A., Quataert, E., & Murray, N. 2005, ApJ, 630, 167
Ward, W. R. 1986, Icarus, 67, 164
Yu, Q., & Tremaine, S. 2002, MNRAS, 335, 965